

2.1 Derivative Basics: Form A Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify.py`). All problems are original; see the notes file for provenance. Prepared with assistance from Claude, an AI model made by Anthropic.

General grading rules

- **Answer-only items** (Q1, Q9, Q13): all or nothing, except where a row below says otherwise.
- **Value vs. derivative.** Giving a derivative where a function value was asked (or the reverse) earns 0 for that part.
- **Show-work items** (Parts II–IV other than Q9): a bare correct answer earns at most half credit.
- **Definition items** (Q6, Q7): the problem states the required method. A correct answer by differentiation rules earns at most 2 of the 8 points.
- **Method is not graded for efficiency.** A correct answer reached by long but valid work gets full credit; the time it costs is the penalty. This matters most in Q2.

Form A

Item	Answer	Rubric / trap
1(a)	1	Read the dot at $(-3, 1)$. <i>Trap</i> : the tangent line drawn there invites the slope, 2; earns 0.
1(b)	-2	Tangent at $(5, 1)$ passes through $(4, 3)$ and $(6, -1)$. <i>Trap</i> : giving $f(5) = 1$.
1(c)	0	Peak at $(-1, 3)$, horizontal tangent. <i>Trap</i> : giving the height 3.
1(d)	DNE	Corner at $(1, 1)$: slopes -2 on the left, 2 on the right. <i>Trap</i> : 0 (“it’s a low point”), or 1 (the height).
1(e)	$(-1, 1)$ and $(3, 6)$	3 pts, all or nothing; union notation is fine. Endpoints may not be included. <i>Trap</i> : intervals where $f < 0$ instead of $f' < 0$, or $(-1, 6)$.
1(f)	$f'(5) < 0 < f'(-3)$	$f'(5) = -2$, $f'(-3) = 2$. 3 pts, all or nothing.
2	$y - 1 = 8(x - 2)$, i.e. $y = 8x - 15$	2 pts $f(2) = 8 - 8 + 1 = 1$; 3 pts the equation. Any correct form. <i>Trap</i> : the slope is printed in the problem. A student who differentiates $x^3 - 4x + 1$ and evaluates at 2 gets 8 again: full credit, but the time is gone. Using 8 as the y -value ($y - 8 = \dots$) earns 2 of 5.
3	$y = 7(x + 1)$, i.e. $y = 7x + 7$	1 pt $r(-1) = -1 - 2 + 3 = 0$; 2 pts $r'(x) = 3x^2 - 4x$; 1 pt $r'(-1) = 3 + 4 = 7$; 1 pt equation in any correct form. <i>Traps</i> : sign slips at a negative input: $(-1)^3 = -1$, $-2(-1)^2 = -2$, $-4(-1) = +4$. The point is $(-1, 0)$, on the x -axis, so $y - 0$ looks odd and students distrust it. A traditional item, deliberately placed after the redundant-slope trap in Q2.
4(a)	5	3 pts. $y = 7 + 3(x - 2) + 2x - 1 = 5x$; the slope is $3 + 2$. <i>Trap</i> : 3 sits in the familiar point-slope position; it earns 0.
4(b)	10	2 pts. y at $x = 2$: $7 + 0 + 4 - 1 = 10$. <i>Trap</i> : 7, read from the left side; earns 0.
5	-2	$16 - 20 + 2$. <i>Trap</i> : p' is printed and invites $p'(2) = 5$; earns 0.
6	$f'(x) = -\frac{3}{(x-2)^2}$	2 pts $f(x+h) = \frac{3}{x+h-2}$, substituted correctly and in parentheses; 2 pts common denominator, numerator $3(x-2) - 3(x+h-2) = -3h$; 2 pts cancel h to get $\frac{-3}{(x+h-2)(x-2)}$; 2 pts let $h \rightarrow 0$. <i>Trap</i> : writing $f(x)+h$ or $\frac{3}{x-2} + h$ for $f(x+h)$, or losing the minus sign across the subtraction. Rules only: at most 2.

Item	Answer	Rubric / trap
7	13	1 pt $g(2) = 10$; 1 pt setup $\frac{x^3 + x - 10}{x - 2}$; 3 pts factoring out $x - 2$ to get $x^2 + 2x + 5$, by any valid method. Synthetic or long division by $x - 2$ works (on 1, 0, 1, -10 , with the zero placeholder). So does grouping with the difference of cubes: $x^3 + x - 10 = (x^3 - 8) + (x - 2) = (x - 2)(x^2 + 2x + 4) + (x - 2) = (x - 2)(x^2 + 2x + 5)$. No particular method is required. 3 pts $4 + 4 + 5 = 13$. <i>Trap</i> : a student who divides and drops the $0x^2$ placeholder gets a wrong quotient. Rules only ($3x^2 + 1$ at 2): at most 2.
8(a)	$-\frac{1}{8}$	$\frac{1}{2} \cdot f'(2)$ for $f(x) = \frac{1}{x}$, $f'(2) = -\frac{1}{4}$. <i>Trap</i> : $-\frac{1}{4}$, ignoring the $2h$; earns 0. Common-denominator algebra also reaches $-\frac{1}{8}$. 3 pts for identifying $f(x) = \frac{1}{x}$ at 2 (or a correct common-denominator setup), 3 pts for the value.
8(b)	-1	$\sin x = \sin x - \sin \pi$, so the limit is the derivative of $\sin$ at π : $\cos \pi = -1$. <i>Trap</i> : “0/0, DNE” or 1; earn 0. 3 pts for recognizing $\frac{\sin x - \sin \pi}{x - \pi}$ as the derivative of $\sin$ at π , 3 pts for $\cos \pi = -1$.
9	(a) 0 (b) 0	3 each. Each function is constant. <i>Traps</i> : (a) $\cos(\pi/4)$; (b) $f'(2) = 4 \cdot 8 - 3 = 29$. In (b), $m(x) = f(2) = 10$ for every x , so $m'(x) = 0$; the problem says “for every x ”.
10(a)	$-3x^{-3/2} + 4 \sin x$	1 pt rewriting $6x^{-1/2}$; 2 pts derivative. <i>Trap</i> : the derivative of $-4 \cos x$ is $+4 \sin x$.
10(b)	$6x^2$	1 pt simplify to $2x^3 - 5$ (for $x \neq 0$); 2 pts. <i>Trap</i> : the fraction bar suggests a quotient rule the class has not learned. Differentiating top and bottom separately ($\frac{8x^3 - 5}{1}$) earns 0.
10(c)	$\frac{10}{3}x^{-1/3} + 6x^{-4}$	1 pt rewriting $5x^{2/3} - 2x^{-3}$; 2 pts. Equivalent radical forms accepted. <i>Trap</i> : $-2 \cdot (-3) = +6$; the exponent $-3 - 1 = -4$.
10(d)	$8x - 12 - 3 \cos x$	1 pt expand $4x^2 - 12x + 9$; 2 pts. <i>Trap</i> : $(2x - 3)^2 \neq 4x^2 + 9$. Students who know the chain rule get the same answer: full credit.
11	$2\pi + 3$	3 pts $f'(x) = 2x - 3 \cos x$; 2 pts value. <i>Trap</i> : $\cos \pi = -1$, so $-3 \cos \pi = +3$; $2\pi - 3$ earns 3.
12	$x = \frac{5\pi}{6}$	2 pts $f'(x) = 1 - 2 \sin x$; 2 pts $\sin x = \frac{1}{2}$; 2 pts keep only $\frac{5\pi}{6}$. <i>Trap</i> : $\frac{\pi}{6}$ is a solution but lies outside $(\frac{\pi}{2}, 2\pi]$; listing it costs 2.
13(a)	II	u falls then rises, turning at $x = -1.5$: u' is a line, negative then positive, crossing zero at -1.5 . <i>Trap</i> : III, the parabola, because u is a parabola.
13(b)	III	v has turning points at -2.5 and 0.5 , rising, falling, rising: v' is $+$, $-$, $+$ with zeros at those x -values. <i>Trap</i> : I, which has the right zeros and the wrong signs.
14	see sketch description	$k' = 2$ for $x < -1$; $k' = 0$ for $-1 < x < 2$; $k' = -2$ for $x > 2$. Open circles at the ends of each segment, at $x = -1$ and $x = 2$. 2 pts per horizontal piece (correct height and interval); -1 once if the open circles are missing or replaced by closed dots. <i>Trap</i> : drawing k 's shape again, or joining the pieces with vertical segments.

Points: $14 + 5 + 5 + 5 + 2 + 8 + 8 + 12 + 6 + 12 + 5 + 6 + 6 + 6 = 100$.